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Using smooth metamers to estimate color appearance metrics for diverse color-normal observers

Kevin A. G. Smet[1] | **Michael A. Webster**[2] | **Lorne A. Whitehead**[3]

[1]Light & Lighting Laboratory, KU Leuven, Ghent, Belgium
[2]Department of Psychology, University of Nevada, Reno, Nevada, USA
[3]Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada

Correspondence
Lorne A. Whitehead, Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, Canada.
Email: lorne.whitehead@ubc.ca

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Abstract

Color-normal subjects sometimes disagree about metameric matches involving highly structured spectral power distributions (SPDs), because their cone fundamentals differ slightly, but non-negligibly. This has significant implications for the design of light sources and displays, so it should be estimated. We propose a broadly applicable estimation method based on a simple adaptive "front-end" interface that can be used with any selected standard color appearance model. The interface accepts, as input, any set of color-matching functions for the individual subject (eg, these could be that person's cone response functions) and also the associated tristimulus values for the test stimulus and also for the reference stimulus (ie, reference white). The interface converts these data into tristimulus values of the form used by the selected color appearance model (which could, eg, be X, Y, Z), while also carrying out the needed transform, which is based on an estimate of the subject's likely previous long-term adaptations to their unique cone fundamentals. The selected standard color appearance model then provides color appearance data that are an estimate of the color appearance of the test stimulus, for that individual subject. This information has the advantage of being interpretable within that model's well-known color space. The adaptive front end is based on the fact that, for any selected input SPD and the subject's unique color matching functions, there can be many different SPDs that are metameric for that individual. Since observer-to-observer color perception differences are minimized for spectrally smooth SPDs, smooth metamers predict color appearances reasonably accurately.

KEYWORDS

adaptation, color appearance, estimation, individual cone fundamentals, individual variation

1 | INTRODUCTION

It is well known that standard color appearance models convert a scene's colorimetric data, based on a Commission Internationale de l'Eclairage (CIE) standard observer, into predicted perceptual attributes such as hue, chroma, and lightness. Although color perceptions cannot be measured directly, they can be estimated through color matching experiments, tests of just noticeable color differences, magnitude estimation, and by asking subject to assess the relative sizes of larger color differences. It is therefore scientifically realistic to evaluate, at least approximately, the accuracy of color appearance models.[1]

This is important for various esthetic and practical reasons. A specific example of growing importance is the need to optimize the spectral power distributions (SPDs) of light used for illumination and in image displays, especially because there are intrinsically conflicting goals of energy efficiency and color accuracy, which necessitate a balancing of those objectives.[2]

For this reason, there is increasing interest in improving the accuracy of color appearance models. In this context, there is growing recognition that, among people who have what is considered normal color vision, there is considerable variability of their cone fundamentals as evidenced, for example, in color matching experiments.[3,4,5]

In general, there certainly are some aspects of the human environment where it is reasonable to make design decisions as if everyone were exactly average, but this is not always appropriate. For example, when selecting the standard height for doorways, it is not sufficient for a door's height simply to exceed that of the *average* person. Yet in other cases, such as selecting the air temperature within a building, selecting the average of individual preferences might make more sense. There is now growing recognition that the design of SPDs for diverse users is a case where averages alone are insufficient.[6,7]

This article begins with a brief summary of a recently reported new method for evaluating the color appearance of surfaces, which was specifically designed to account for the natural variation of cone fundamentals among human observers.[8] That model was based on one we recently developed to derive uniform color metrics from principles of efficient coding.[9] In the new adaptive model, we showed that the numerical coefficients could be automatically adjusted to maintain a constant level of color contrast within a sample set selected, with uniform hue spacing, from the Munsell system. In particular, we determined that this automatic adjustment would largely counteract color appearance changes that would otherwise arise from variations in individual cone fundamentals. This approach yielded results that matched well with previous experimental observations. Put simply, it is not possible to accurately estimate color perceptions of nonstandard individual observers by simply inserting, into standard colorimetric formulas, their unique personal color matching functions, because that ignores these important known adaptation effects.

However, there are two important factors that limited the utility of the approaches just mentioned for predicting color appearances for individual observers: (a) the simple color appearance model used in that case was not designed to account for a number of important visual effects (such as the Hunt effect), which are modeled in more complete color appearance models, such as CIECAM02.[10] (b) The model was based on a distinct color metric and thus did not provide values in terms of the standard color appearance metrics already in widespread use, which help to maintain shared understanding about color throughout research and industry. Therefore, it would be preferable to incorporate models of adaptation effects into existing, already-known color appearance models.

Toward that end, we present here a simple approach that addresses those two factors. It is based on the concept of a smooth metamer for color translation, in the form of a very smooth SPD that is adjusted so as to be metameric with the test SPD, for the selected test observer.

Finally, we show that implementation of a smooth metamer translation can be exactly emulated by a matrix multiplication of the input observer's test SPD tristimulus values (calculated using their color matching functions or cone fundamentals) by an adaptation matrix that is defined by a simple formula based on the primary components of the smooth metamer SPD, the test observer's color matching functions, and the color appearance model's standard color matching functions.

2 | SMOOTH METAMERS FOR COLOR TRANSLATION

Using the test observer's color matching functions, it is straightforward to determine whether a given SPD is metameric, for that observer, with any selected test SPD. In some cases, functional relationships are not invertible in closed form, so an iterative procedure is required to find such an SPD. As an example, a simple approach could be to begin with an arbitrary function based on three parameters. For any set of values for those parameters, the tristimulus values of the resultant function can be calculated, and therefore the required values could be determined by iteratively adjusting them to make the new function's tristimulus values match those of the selected test SPD.

While simple conceptually, an iterative approach can be impractical from a computational perspective, even with today's readily available computation power, because in multivariable computational optimization, it is commonplace to require millions of steps, which becomes impractical if each step in the iteration includes an iterative loop that might also consist of millions of steps. We therefore used a procedure for determining the metameric match that is noniterative and also familiar to many—making the matching function a linear combination of three fixed "primary" SPDs, which allows the metamers to be evaluated by means of fast and deterministic matrix calculation. For the primaries, we selected three simple functions, to which we assigned the labels $R(\lambda)$, $G(\lambda)$, and $B(\lambda)$ (for red, green, and blue). They are described by simple formulas that satisfy the following four requirements: (a) the functions, and their slopes, are everywhere continuous; (b) individually, they are well localized in the blue, green, and red regions of the spectrum in order to enable a wide gamut of additive colors; (c) when mixed in equal quantity, they add to a constant value for all wavelengths; and (d) more generally, they blend to produce smooth spectra that are free of spectrally localized features.

We were able to devise formulas that simultaneously satisfy these requirements. The proposed ones are most simply expressed in terms of a dimensionless measure of wavelength, defined in Equation (1).

$$x = \frac{(\lambda - 535\text{nm})}{80\text{nm}} \tag{1}$$

This dimensionless parameter x applies for any value of wavelength. We have selected different formulas for five different ranges of values of x, as depicted in the top row of Table 1. Together, these ranges span all possible wavelengths. Note that at this stage, these bands have no particular relationship to various choices of wavelengths used for various CIE colorimetric formulas; rather these formulas are simply smooth primary functions of wavelength. The formulas are depicted in the three subsequent rows of Table 1 and graphed in Figure 1. The graph shows that both the values and the slopes are continuous, as would be expected for a physically plausible function (Note also that the only reason that Figure 1 plots over the values range from −1.5 to 1.5 is that beyond that range those particular functions are all constant. That range of x-values corresponds to wavelengths from 415 nm to 655 nm, which has no significance from the perspective of subsequent radiometric calculations, which use a standard CIE wavelength range.)

We note that piece-wise quadratic definitions of distributions have been employed in other contexts where their smooth blending characteristics have been deemed valuable.[11,12] We believe that they are particularly useful in this context because, as described subsequently, it is a simple fast calculation to find a linear combination of the three smooth primary functions that will be metameric with almost any real spectral power distribution. Of course the generated metamer will have a different SPD than the one selected for matching, but interestingly, they are often quite similar, presumably because many common SPDs are themselves quite smooth.

As an example, Figure 2 shows a fairly smooth SPD and a metameric match (for the CIE standard 10° reference observer) based on a linear combination of the three smooth primary functions.

Another way to appreciate the value of these three primaries in creating smooth metamers is displayed in Figure 3, which shows a smooth, natural looking variation of the resultant SPDs as a function of their hue.

3 | APPLYING SMOOTH METAMERS TO COLOR TRANSLATION

With the help of these primaries, it is fairly simple to construct multiple metameric SPDs that can then be evaluated from a color-constancy perspective among observers with differing cone fundamentals. As a first step, 200 random SPDs were generated as follows: the first step was to add together six randomly generated Gaussian peaks, each defined by a central wavelength, a *SD* and a peak intensity. For each SPD created, the six peak Gaussian wavelengths were randomly picked between 450 and 600 nm, the 6 *SD* were randomly selected between 5 and 50 nm, and the peak intensities were randomly selected

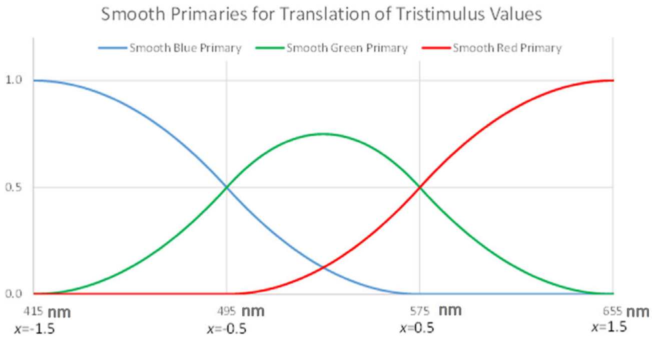

FIGURE 1 The smooth primary functions $B(\lambda)$, $G(\lambda)$, $R(\lambda)$ as defined in Table 1 and Equation (1). The three functions and their derivatives are continuous and they add to one everywhere

TABLE 1 Primary formulas based on x as defined in Equation (1)

	$x < -1.5$	$-1.5 < x < -0.5$	$-0.5 < x < 0.5$	$0.5 < x < 1.5$	$1.5 < x$
$B(x)$	1	$(-0.5x^2 - 1.5x - 0.125)$	$(0.5x^2 - 0.5x + 0.125)$	0	0
$G(x)$	0	$(0.5x^2 + 1.5x + 1.125)$	$(-1.0x^2 + 0.75)$	$(0.5x^2 - 1.5x + 1.125)$	0
$R(x)$	0	0	$(0.5x^2 + 0.5x + 0.125)$	$(-0.5x^2 + 1.5x - 0.125)$	1

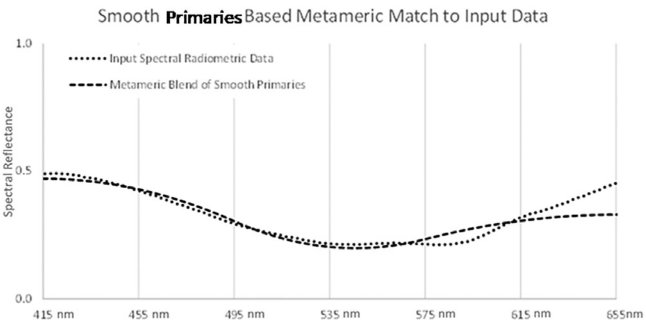

FIGURE 2 A metameric blend of the three primaries B, G, R approximating an input spectral radiometric distribution

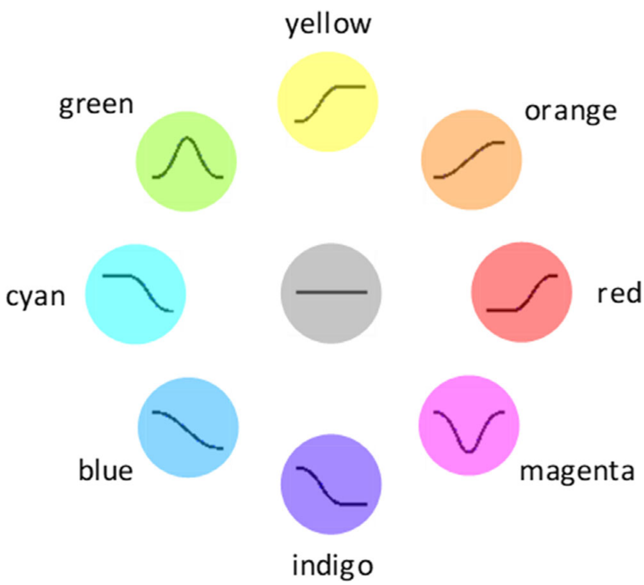

FIGURE 3 Graphs of intensity vs wavelength (in colored circles) for various linear combinations of the three primaries defined in Table 1, demonstrating their ability to provide smooth SPDs for all hues. Beginning with red and going counterclockwise, the RGB coefficients were: (1,0,0); (1,0.5,0); (1,1,0); (0,1,0); (0,1,1); (0,0.5,1); (0,0,1); (1,0,1). The neutral gray has RGB coefficients of (0.5,0.5,0.5)

between 0 and 1. At that stage, the sum of those six Gaussian functions would generally not be achromatic. The reason for positioning the Gaussian peaks in the range from 450 nm to 600 nm is that this is the range in which the cone responses differ from one another substantially.

The next step was to modify them to make them achromatic, which was done by adding the necessary amount of the B, G, and R primaries, so that the tristimulus values matched those for an SPD with a uniform spectral intensity of 0.5, for a standard observer. The resultant 200 gray SPDs are depicted in Figure 4. (The selected range of wavelengths in the chart shows most of the variations of these SPDs, for clarity, but all calculations were done in the range 380–780 nm, using 1 nm bands.)

Using the Asano model,[3] we selected 10 color matching functions sets (categorical observers) that differed from the standard observer in a manner that reflected typical individual variations among 10° color-normal observers. We then calculated the resultant chromaticity, evaluated in the standard $u'v'$ metric, for each of the 200 SPDs, for each of the 10 subjects.

As an additional characterization of each of the 200 SPDs, we calculated three different aspects of their variation. The first was simply the intrinsic “nonflatness” of the SPD, which was calculated as the SD of the SPD value over the range from 450 to 600 nm. The second was the *SD* of the u' values for the 10 observers for that SPD, and the third was corresponding *SD* for the v' values. Figure 5 plots the u' *SD* for each SPD against its intrinsic nonflatness, and Figure 6 does the same for the v' values. These plots demonstrate that generally, the “smoother” the SPD, as defined by a lower *SD*, the lower would be the range of color metric differences between the diverse observers.

In considering both Figures 5 and 6, we note that the calculated deviations were expected to be small (noting that the JND near the achromatic point is considered to be roughly 0.0013.[13]) Therefore, in this case, we believe that the impacts of nonlinearities in actual perception of color differences are small and that the trends depicted in the two plots are real. It is also interesting to note that the mean slope in Figure 6 is about twice as large as in Figure 5. Presumably this arises from a combination of the differing spacing of the peaks of the cone fundamentals and the parameters within the formula for u' and v'; we do not ascribe any particular significance to this ratio.

We also note that, on the topic of generating useful metamers, researchers have occasionally studied a concept termed “fundamental metamers.”[14,15] In that context, the fundamental metamer was the linear combination of the X, Y, and Z color matching functions, calculated using Cohen's R-matrix, that was metameric with a selected SPD. A more modern version of that same idea would be to use the cone fundamentals in an analogous manner. Neither of those approaches works well for the purpose described here, because often this yields negative values at some wavelengths and the resultant SPDs differ considerably from commonly occurring SPDs. As an obvious example, most natural achromatic SPDs are fairly flat and this is not true for fundamental metamers.

The fact that smooth SPDs yield reduced color discrepancies among diverse individuals justifies the use of smooth metameric SPDs as color appearance

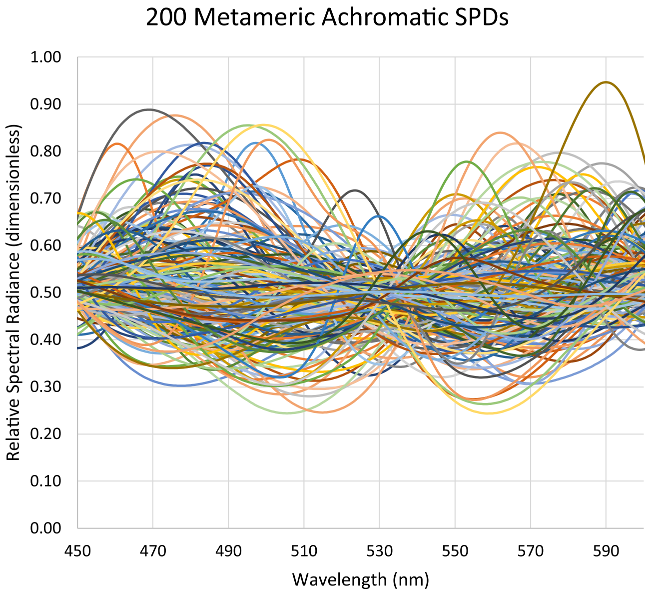

FIGURE 4 The 200 SPDs that are metameric to an SPD of constant value 0.5, for the standard 10° observer

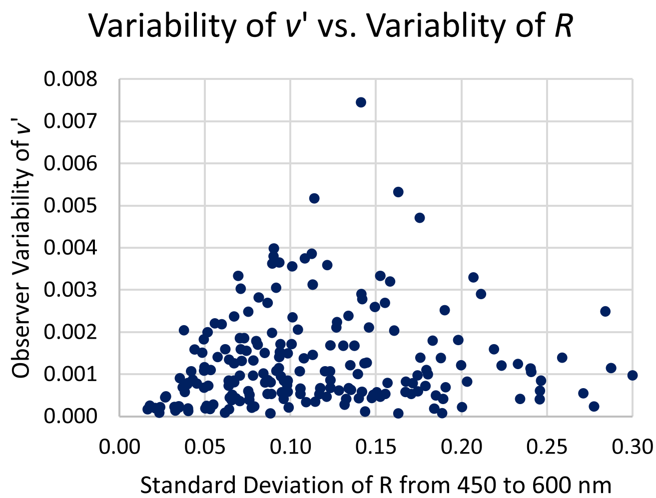

FIGURE 6 A plot of observer variability of *v'* (among 10 random diverse observers) vs SD for 200 randomly generated metameric SPD. Responses to smooth SPDs are more stable between diverse observers

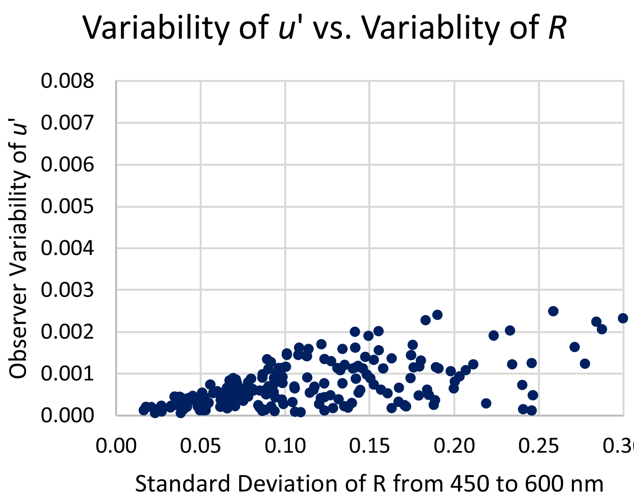

FIGURE 5 A plot of observer variability of *u'* (among 10 random diverse observers) vs *SD* for 200 randomly generated metameric SPD. As expected, responses to smooth SPDs are very stable between diverse observers

"translators" between different observers. To clarify, the process would be as follows: (a) take the SPD of a test spectral stimulus for a given individual user, as well as the SPD of the reference (white) spectral stimulus, and also the individual's color matching functions. Based on that information, generate two spectrally smooth metamers, for that observer, with the first one matching the test stimulus and the second one matching the reference stimulus. (b) Input those two "translation" stimuli into a standard color appearance model, in order to determine the corresponding color appearance values from that model, which then serve as a useful estimate for the color appearance metric for that individual. We next show that these steps can be emulated by means of a simple matrix-based calculation, which yields the same mathematical result without requiring the separate linear combination steps just mentioned.

4 | A SIMPLE MATRIX-BASED APPROACH EMULATES TRANSLATION

To assist this description, it will be helpful to note that everything stated above is equally true regardless of the form of color matching functions used. The three most commonly used ones are as follows:

$\bar{r}(\lambda), \bar{g}(\lambda), \bar{b}(\lambda)$ with tristimulus values being R, G, B.
$\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$ with tristimulus values being X, Y, Z.
$\bar{l}(\lambda), \bar{m}(\lambda), \bar{s}(\lambda)$ with tristimulus values being L, M, S.

For a given individual, these are generally close to being linear combinations of one another. The procedure described below is very robust and, interestingly, it works for any linear combination of these functions, as a result of the in-built compensation provided by matrix inversion. For this reason, here we will use the following general symbols to refer to any of these different representations of an observer's cone fundamentals and also for the associated tristimulus values:

$\bar{f}_1(\lambda)$,$\bar{f}_2(\lambda)$,$\bar{f}_3(\lambda)$ with tristimulus values being F_1,F_2,F_3.

More specifically, we will be interested in a specific standard set used in the selected color appearance model, denoted by the suffix S, while the individual observer's set is be denoted by the suffix I:

$\bar{f}_{1S}(\lambda)$,$\bar{f}_{2S}(\lambda)$,$\bar{f}_{3S}(\lambda)$ with tristimulus values being F_{1S},F_{2S},F_{3S}.
$\bar{f}_{1I}(\lambda)$,$\bar{f}_{2I}(\lambda)$,$\bar{f}_{3I}(\lambda)$ with tristimulus values being F_{1I},F_{2I},F_{3I}.

In all cases, the standard integral relation in Equation (2) calculates tristimulus values.

$$F_n = \int_0^\infty K\overline{f_n}(\lambda)E_{e,\Omega}(\lambda)\mathrm{d}\lambda \tag{2}$$

In Equation (2) $E_{e,\Omega}(\lambda)$ is the spectral radiance of either the stimulus or the reference white, $n = 1$, 2, or 3 and K is a constant that, for the purposes of these calculations, will cancel out and will therefore be omitted for simplicity in the treatment below.

At this point if might be helpful to point out that this approach makes no assumption that relates the SPD of the reference white to that of the stimulus. For example, the stimulus could be a self-luminous object.

We now wish to mathematically consider three primary functions, R(λ), G(λ), and B(λ) that will be added together to comprise a smooth SPD that will be adjusted to be metameric with the test SPD and again, with the reference SPD. For the individual observer, the matrix depicted in Equation (3) characterizes the relationship between these primary functions and color matching functions. This can be thought of as a "sensitivity" matrix, with each element including one of the three primary functions and one of the three color-matching function. Thus, there are nine permutations in this 3 × 3 matrix. Note that the rows of this matrix are the F_{iI} tristimulus values of the three primary SPDs for the individual observer.

$$\mathbf{M_I} = \begin{vmatrix} \int_0^\infty B(\lambda)\bar{f}_{1I}(\lambda)\mathrm{d}\lambda & \int_0^\infty B(\lambda)\bar{f}_{2I}(\lambda)\mathrm{d}\lambda & \int_0^\infty B(\lambda)\bar{f}_{3I}(\lambda)\mathrm{d}\lambda \\ \int_0^\infty G(\lambda)\bar{f}_{1I}(\lambda)\mathrm{d}\lambda & \int_0^\infty G(\lambda)\bar{f}_{2I}(\lambda)\mathrm{d}\lambda & \int_0^\infty G(\lambda)\bar{f}_{3I}(\lambda)\mathrm{d}\lambda \\ \int_0^\infty R(\lambda)\bar{f}_{1I}(\lambda)\mathrm{d}\lambda & \int_0^\infty R(\lambda)\bar{f}_{2I}(\lambda)\mathrm{d}\lambda & \int_0^\infty R(\lambda)\bar{f}_{3I}(\lambda)\mathrm{d}\lambda \end{vmatrix} \tag{3}$$

Similarly, the matrix in Equation (4) characterizes the relationship between these primary functions and the standard color matching functions used in the chosen color appearance model. Again, note that the primary SPDs' standard observer tristimulus values F_{iS} are in the rows.

$$\mathbf{M_S} = \begin{vmatrix} \int_0^\infty B(\lambda)\bar{f}_{1S}(\lambda)\mathrm{d}\lambda & \int_0^\infty B(\lambda)\bar{f}_{1S}(\lambda)\mathrm{d}\lambda & \int_0^\infty B(\lambda)\bar{f}_{1S}(\lambda)\mathrm{d}\lambda \\ \int_0^\infty G(\lambda)\bar{f}_{2S}(\lambda)\mathrm{d}\lambda & \int_0^\infty G(\lambda)\bar{f}_{2S}(\lambda)\mathrm{d}\lambda & \int_0^\infty G(\lambda)\bar{f}_{2S}(\lambda)\mathrm{d}\lambda \\ \int_0^\infty R(\lambda)\bar{f}_{3S}(\lambda)\mathrm{d}\lambda & \int_0^\infty R(\lambda)\bar{f}_{3S}(\lambda)\mathrm{d}\lambda & \int_0^\infty R(\lambda)\bar{f}_{3S}(\lambda)\mathrm{d}\lambda \end{vmatrix} \tag{4}$$

Lastly, the observer's tristimulus values for the test SPD can be expressed in the row vector depicted in Equation (5), using the usual integrals.

$$\mathbf{V_I} = \left| \int_0^\infty \bar{f}_{1I}(\lambda)E_{e,\Omega}(\lambda)\mathrm{d}\lambda,\ \int_0^\infty \bar{f}_{2I}(\lambda)E_{e,\Omega}(\lambda)\mathrm{d}\lambda,\ \int_0^\infty \bar{f}_{3I}(\lambda)E_{e,\Omega}(\lambda)\mathrm{d}\lambda \right| \tag{5}$$

We now wish to convert these to adapted tristimulus values, for use as input to a color appearance model that is based on the color matching functions for the standard observer. This conversion can be carried out in the manner shown by means of the simple matrix multiplication depicted in Equation (6), which can be proven to be correct by substitution.

$$\mathbf{V_{IS}} = \mathbf{V_I}\boldsymbol{M}_\mathbf{I}^{-1}\boldsymbol{M}_\mathbf{S} \tag{6}$$

Examining the form of Equation (6), it is clear that if the two sets of color matching functions are identical, then $\boldsymbol{M}_\mathbf{I}^{-1}\boldsymbol{M}_\mathbf{S}$ is simply the identity matrix, as would be expected in that case.

There is an additional valuable characteristic of Equation (6) that some may find counterintuitive—the exact same result is obtained for $\mathbf{V_{IS}}$ when the input vector is obtained using *any* linear combination of the test observer's color matching functions. In other words, it does not matter whether the functions describing the test observer are in the format $\bar{x}(\lambda)$,$\bar{y}(\lambda)$,$\bar{z}(\lambda)$, or $\bar{l}(\lambda)$,$\bar{m}(\lambda)$,$\bar{s}(\lambda)$, or $\bar{r}(\lambda)$,$\bar{g}(\lambda)$,$\bar{b}(\lambda)$, or any other linear combination of the test observer's cone fundamentals. Nor does the order in which those functions are entered into the formula matter at all—all of those variants are accounted for by the combined mathematical processing of Equations (3)–(6).

From this perspective, Equation (6) is a universal translator that enables *any* standard color appearance model to accept tristimulus values based on *any* color matching function set for *any* color-normal individual observer. The only information needed by this "front end adaptor" is (a) the SPD for the test and white stimuli,

(b) some linear combination of the individual observer's cone fundamental functions, and (c) the color matching functions that are used within the selected color appearance model. Equation (6) then produces the proper tristimulus values for input into that color appearance model, in order to yield the best estimate for the individual's expected color perception, in reference to the standard color space of that color appearance model.

An Excel spread sheet is available from the authors to carry out this adaptation to provide input to the standard CIE colorimetric calculations such as for x, y and u', v' and for color appearance models such as CIELAB L^*, a^*, b^* and CIECAM02. The method has also been implemented in Luxpy,[16] a Python package for color science calculation developed by one of the authors.

5 | IMPLICATIONS OF AN ADAPTIVE FRONT END FOR COLOR APPEARANCE METRICS

An important consideration is that the smooth metamer translation procedure described here is only an approximation, albeit one that yields predicted values for the color appearance metric of an individual observer that are likely to be more accurate than would be the case if adaptation were ignored. We also note that the previously mentioned adaptive model[8] probably does not perfectly coincide with the simple calculation shown here, although they are comparable in terms of the number of adjustable parameters employed (eight in the previous model and nine in this one).

These considerations motivated a comparison between the two calculation systems, to help determine if they make similar predictions. As an extreme stress test, we compared these predictions for an observer who would fall outside the range of normal color vision. For this purpose, the selected test observer was one for which the spacing between the L and M cones was half of that of the standard observer (corresponding to a mild anomalous trichromat.) Figure 7 depicts the color shifts that would be calculated without using either color adaptation formula, for a selected set of 20 Munsell reflective samples equally spaced for hue. In this figure, as well as in Figures 8 and 9, a distance of 1 unit in the plane corresponds to the magnitude of a chroma increase of one unit of chroma within the Munsell system, while holding hue and value constant, with the vertical direction corresponding to the direction for chroma increases for the hue 5Y. These shifts could equally have been presented in other color spaces; the Munsell space was selected since it is popular and was designed to be as perceptually uniform as possible, based on available data at the time.

Next, the adaptive model as described in the literature[8] was applied to predict the perceived color contrasts of the samples. Figure 8 shows that this reduces the predicted deviations to near the limits of visual perception. Specifically, the model predicts that even for the extreme observer we considered, adaptation should normalize their color perception such that the range of color contrasts they experience should be comparable to the standard observer. We emphasize that the models we propose are not designed to correctly predict the color metrics for extreme variations in color vision and in particular should not be used to characterize color experience of individuals with color deficiencies. However, recent studies have found that these experiences for anomalous trichromats do tend to be more similar to color normals than expected from their cone sensitivities, which is in the correct direction, if not the correct magnitude, predicted by adaptation to their deficiency.[17]

Lastly, we restored the errors shown in Figure 7 and then instead applied the front-end transform matrix $\boldsymbol{M}_t^{-1}\boldsymbol{M}_s$ as depicted in Equation (6) to adapt the tristimulus values for both the white standard and the Munsell samples. For the extreme observer we considered, the matrix took the form shown in Equation (7).

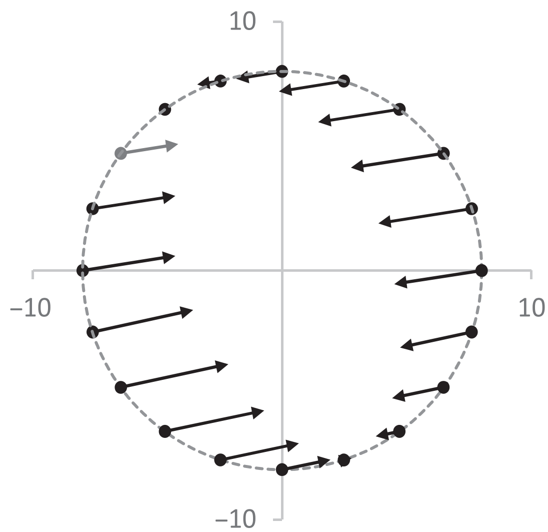

FIGURE 7 The calculated color shifts, (without applying any color contrast adaptation formula), when switching from standard cone fundamentals to abnormal cone fundaments with the L-M separation reduced by 50%. The colors were selected Munsell colors, the scale is in Munsell units, with hue 5Y aligned with the positive y-axis

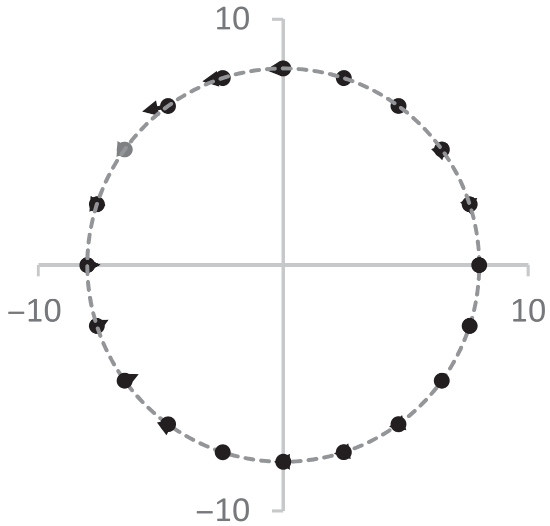

FIGURE 8 The color shifts calculated as in Figure 7, but, after implementation of higher-level automatic color contrast adaptation as in Equation (8)

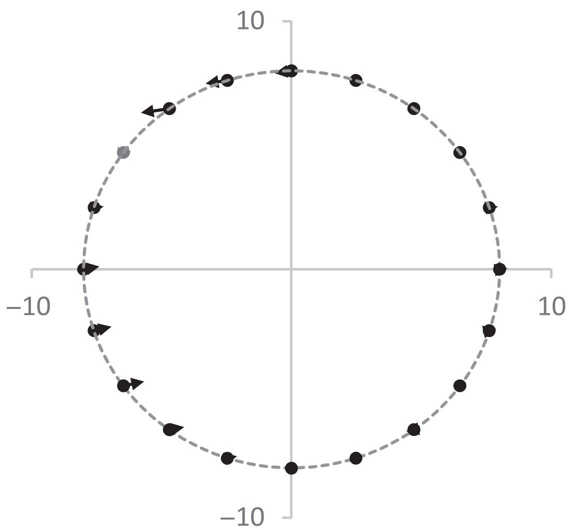

FIGURE 9 The calculated color shifts as in Figure 8, without implementation of the full adaptation model, but instead with the “front-end” adaptation of the input stimulus functions according to Equation (6)

$$\boldsymbol{M}_t^{-1}\boldsymbol{M}_s = \begin{vmatrix} \mathbf{2.00} & \mathbf{0.00} & \mathbf{0.00} \\ \mathbf{-1.20} & \mathbf{1.00} & \mathbf{0.00} \\ \mathbf{0.00} & \mathbf{0.10} & \mathbf{1.00} \end{vmatrix} \tag{7}$$

It is interesting that the matrix values in Equation (7) differ significantly from the identity matrix—primarily in the top two values in the left column. Effectively, in this case, the matrix creates a “new L cone” that is a linear combination of the L and M cones, which is shifted to coincide with the standard L cone location. The calculated result is shown in Figure 9.

This completely different approach for emulating adaptation yields values nearly identical to the calculated values depicted in Figure 8.

As an additional test of the stability and robustness of the new front-end method for estimating individual color metrics, we also evaluated the predictions for three very different primary functions 1, 2, 3, respectively, defined by Equations (8)–(10) and illustrated in Figure 10.

$$\begin{gathered} \text{If } -1.5 < x < 1.5 \text{ then } P_1(x) = 0.5 + 0.5 * \cos\left(\frac{2\pi}{3}x\right) \\ \text{Otherwise } P_1(x) = 0 \end{gathered} \tag{8}$$

$$P_2(x) = x = 1 \tag{9}$$

$$\begin{gathered} \text{If } -1.5 < x < 1.5 \text{ then } P_3(x) = 0.5 + 0.5 * \sin\left(\frac{\pi}{3}x\right) \\ \text{Otherwise } P_3(x) = 0.5 + \frac{0.5x}{|x|} \end{gathered} \tag{10}$$

(Note that these functions satisfy neither criterion ii, nor criterion iii, as described in Section 2 above, so they would not be desirable primaries; they are tested here as an extreme case for evaluating the sensitivity of that adaptation matrix of Equation (6) to the detailed features of the selected primary set.) In this case, the calculated values for the adaptation are very similar to those in Equation (7), as shown in Equation (11).

$$\boldsymbol{M}_t^{-1}\boldsymbol{M}_s = \begin{vmatrix} \mathbf{2.02} & \mathbf{0.00} & \mathbf{0.00} \\ \mathbf{-1.22} & \mathbf{1.00} & \mathbf{0.00} \\ \mathbf{-0.04} & \mathbf{0.00} & \mathbf{1.00} \end{vmatrix} \tag{11}$$

We tried the matrix shown in Equation (11), instead of the one shown in Equation (7), to predict the expected

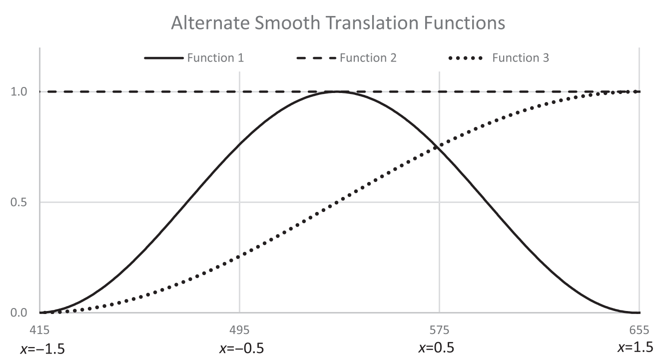

FIGURE 10 An example of an alternate set of smooth spectral power distributions functions, for the purpose of testing the degree of sensitivity of the calculation to the specific choice of smooth primary functions

color appearance coordinates in Figure 9, and the results were very similar, which is not surprising given the similarity of the two matrices. This suggests that the precise details of the smooth primary functions are not critical—they simply need to be fairly smooth and substantially linearly independent. Nevertheless, they should be standardized, with an eye to convenience and practicality in their use. The format recommended in Table 1 is suggested because it satisfies all four of the aforementioned criteria of continuity of value and slope, large gamut, fairly uniform achromatic SPDs and very smooth blending. It is also an advantage that most common color samples can be modeled using purely positive coefficients for these three primaries.

6 | CONCLUSION

Adaptation effects are important to incorporate when predicting color metrics for individual observers, because they tend to undo the effects of variations in spectral sensitivity of observers' color matching functions. We have demonstrated a simple practical approach for doing this, which accepts as input the SPD of a test stimulus and reference white as well as the individual's unique color matching functions (in any format). The adaptation calculation is carried out in a simple, direct matrix calculation that exactly emulates the smooth metamer translation concept described here. The generality of this method is very convenient, enabling the use of *any* format of *any* set of color matching functions to describe the individual observer, and *any* color appearance model that was designed for a specified standard observer. Of course, the precise predictions can only be viewed as approximations, because the use of different smooth metamers yields slightly different predictions. Thus, the primary consequence of the adaptive transformation is to remove much of the difference among individuals expected from their different cone sensitivities, while still preserving inter-observer differences in metamerism. This may yield more accurate assessments of the impact of normal variations in color matching functions for color appearance metrics, which could help guide the development of spectrally complex displays and lighting systems that will provide more accurate color experiences for most normal observers. The model also makes predictions for extreme variations (eg, color deficiencies) that are in qualitative agreement with evidence for compensatory adjustments for color deficiencies. However, more empirical work is necessary for evaluating these compensations before the model can be extended or should be used to evaluate color appearance metrics for individuals with color deficiencies.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

ORCID

Kevin A. G. Smet https://orcid.org/0000-0003-3825-6274
Lorne A. Whitehead https://orcid.org/0000-0002-4170-0033

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AUTHOR BIOGRAPHIES

Kevin A. G. Smet obtained a PhD in engineering at KU Leuven, Ghent, Belgium, in 2011. He is currently appointed as an associate research professor at KU Leuven. Kevin is the team leader of the Appearance & Perception Group of the Light & Lighting Laboratory and is an active member of CIE Division 1. His research interests include color rendition and color quality of white light sources, color appearance modeling, memory and preferred colors, perception of light and color in virtual reality, and color science in general.

Michael A. Webster is Professor of Psychology at the University of Nevada, Reno, USA, where he directs the university's Center for Integrative Neuroscience (an NIH COBRE grant) and co-directs the graduate and undergraduate neuroscience degree programs. His research interests focus on color and spatial vision, individual differences, and how vision adapts to properties of the environment and the observer.

Lorne A. Whitehead is Professor of Physics at the University of British Columbia, Vancouver, Canada, where he is also the Special Advisor on Entrepreneurship, Innovation & Research. His research interests include improving the quality of color perception in information displays and artificially illuminated spaces. He often contributes to CIE and IES committees, and some of his inventions are applied in modern televisions, computers, and lighting products.

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